CONVERGENCE OF INFINITE PRODUCTS OF MATRICES AND INNER–OUTER ITERATION SCHEMES

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Dedicated to Wilhelm Niethammer on the occasion of his sixtieth birthday.

Abstract. We develop conditions under which a product \( \prod_{i=0}^{\infty} T_i \) of matrices chosen from a possibly infinite set of matrices \( S = \{T_j | j \in J\} \) converges. We obtain the following conditions which are sufficient for the convergence of the product: There exists a vector norm such that all matrices in \( S \) are nonexpansive with respect to this norm and there exists a subsequence \( \{i_k\}_{k=0}^{\infty} \) of the sequence of the nonnegative integers such that the corresponding sequence of operators \( \{T_{i_k}\}_{k=0}^{\infty} \) converges to an operator which is paracontracting with respect to this norm. We deduce the continuity of the limit of the product of matrices as a function of the sequences \( \{i_k\}_{k=0}^{\infty} \). But more importantly, we apply our results to the question of the convergence of inner–outer iteration schemes for solving singular consistent linear systems of equations, where the outer splitting is regular and the inner splitting is weak regular.

Key words. iterative methods, infinite products, contractions.

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